MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2015/2016

BSP2014 – INTRODUCTION TO APPLIED PROBABILITY AND STOCHASTIC PROCESSES

(All sections / Groups)

2 MARCH 2016
2.30 p.m. – 4.30 p.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of **TWO (2)** printed pages exclusive of the cover page, formulae sheet and statistical table.

2. Answer **ALL** four questions in the answer booklet provided.

3. Students are allowed to use authorized calculators by lecturer only.

4. Marks are shown at the end of each question.

Question 1 [31 marks]

(a) Given X as a random variable with the probability density function $f(x) = \frac{c}{15}x^3$ for $1 \le x \le 2$.

i) Determine the value of c. [4 marks]
ii) Find P (X > 1.8). [4 marks]
iii) Find the mean and variance for X. [11 marks]
iv) Find E ($2X^2 + 3X - 5$). [4 marks]

(b) The number of students seeking information regarding credit transfer at FOM's counter is at the rate of 0.1 per day. Let N(t) denote the number of students seeking information regarding credit transfer at the counter after t days from the first day of the trimester.

i) Determine the pmf for N(t). [1 mark]
ii) Find the probability of 4 students after 7 days. [3 marks]
iii) Find the probability of no student asking during weekdays in week 1. [4 marks]

Question 2 [35 marks]

Given X and Y have the following joint density function $f(x,y) = \frac{6-x-y}{8}$ for $0 < x < 2 ; 2 < y < 4$.

(a) Find marginal probability density function for X and Y. [10 marks]
(b) Find the joint cumulative density function. [8 marks]
(c) Find P(2 < Y < 3 | X = 1). [6 marks]
(d) Is X and Y independent? Prove it. [3 marks]
(e) Determine the moment generating function for X. [8 marks]

Continued..

Question 3 [17 marks]

(a) In testing a certain kind of truck tire over rugged terrain, it is found that 25% of the trucks fail to complete the test run because of a blowout. Of the next 8 trucks tested,

i) find the probability that 3 or 6 trucks have blowout. [5 marks]

ii) find the probability that at least 2 trucks have blowout. [5 marks]

(b) The length of time between breakdowns of an essential piece of equipment is important in the decision of the use of auxiliary equipment. It is known that the mean time between breakdowns of a generator is 15 days.

i) If the generator has just broken down, what is the probability that it will break down within the next 21 days? [4 marks]

ii) What is the probability that the generator will operate for 30 days without a breakdown? [3 marks]

Question 4 [18 marks]

On a given day, Emmett decide how to go to work either drives to work, takes the train or take a bus. Suppose that X_n is a Markov Chain depending on how Emmett went to work the previous day, the probability of choosing any one of the means of transportation is given by the following transition probability matrix:

$$P = \begin{bmatrix} 1/6 & 2/3 & 1/6 \\ 1/2 & 1/3 & 1/6 \\ 2/5 & 1/2 & 1/10 \end{bmatrix}$$

(a) Given that Emmett took the train on Monday, what is the probability that he will not take the train to work on Wednesday? [11 marks]

(b) Suppose that the initial chances for Emmett to select each of the transportation mode is 20%, 50% and 30% respectively. What are the chances for each transportation mode in next 2 days? [7 marks]

End of Page

FORMULAE

A. PROBABILITY DISTRIBUTION

Bernoulli Probability Distribution

$$P(X = x) = p^x q^{1-x} \qquad \text{for } x = 0, 1$$

Binomial Probability Distribution

$$P(X = x) = {}^nC_x \, p^x q^{n-x} \qquad \text{for } x = 0, 1, \ldots, n$$

Poisson Probability Distribution

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!} \qquad \text{for } x = 0, 1, 2, \ldots$$

Geometric Probability Distribution

$$P(X = x) = pq^{x-1} \qquad \text{for } x = 1, 2, \ldots$$

Uniform Probability Distribution

$$f(x) = \frac{1}{b-a} \qquad \text{for } a < x < b$$

Exponential Probability Distribution

$$f(x) = \lambda e^{-\lambda x} \qquad \text{for } x > 0$$

Normal Probability Distribution

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \qquad \text{for } -\infty < x < \infty$$

Gamma Probability Distribution

$$f(x) = \frac{x^{\alpha-1} e^{-x/\beta}}{\beta^\alpha \Gamma(\alpha)} \qquad \text{for } x \geq 0$$

where $\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx$ for $\alpha \geq 0$

with $\Gamma(\alpha) = (\alpha - 1)\,\Gamma(\alpha - 1)$

with $\Gamma(n) = (n - 1)!$ when n is a positive integer.

Chi-Square Probability Distribution

$$f(x) = \frac{x^{\left(\frac{n}{2}-1\right)} e^{-x/2}}{2^{\frac{n}{2}}\Gamma\left(\frac{n}{2}\right)} \qquad \text{for } x \geq 0$$

B. MOMENT

k^{th} Moment about the origin of X:

$$\mu'_k = E[X^k]$$

k^{th} Moment about the mean of X:

$$\mu_k = E[X-\mu]^k$$

Moment Generating Function for X:

$$M_X(t) = E\left(e^{tx}\right)$$

Derivative of Moment Generating Function for X:

$$M_X^{(r)}(t) = E\left(X^r e^{tx}\right)$$

C. STOCHASTIC PROCESS

Simple Random Walk:

$$P(X_n = m) = \begin{pmatrix} n \\ \frac{n+m}{2} \end{pmatrix} p^{(\frac{n+m}{2})} q^{(\frac{n-m}{2})}$$ where m and n are both even or both odd.

$E(X_n) = n(p-q)$; $Var(X_n) = 4npq$

Gambler's Ruin Problem

$$P_a = \begin{cases} \dfrac{(\frac{q}{p})^a - (\frac{q}{p})^c}{1-(\frac{q}{p})^c}, & p \neq q \\ 1-\dfrac{a}{c}, & p = q \end{cases} \quad ; \quad P_b + P_a = 1$$

$$D_a = a(c-a) \quad \text{if} \quad p = q$$

$$D_a = \frac{1}{q-p}\left\{a - c\left[\frac{1-\left(\frac{q}{p}\right)^a}{1-\left(\frac{q}{p}\right)^c}\right]\right\} \quad \text{if} \quad p \neq q$$

D. POISSON PROCESS

Pmf for N(t) : $$P\{N(t)=n\}=\frac{e^{-\lambda t}(\lambda t)^n}{n!}$$
Pdf for T_n: $$f(t)=\lambda e^{-\lambda t}$$
Pdf for S_n : $$f(t)=\frac{\lambda e^{-\lambda t}(\lambda t)^{n-1}}{(n-1)!}$$